Dual Reflector Shaping, A Wider Range Of Solutions For SKA Antennas

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Introduction
Traditionally, reflector antennas have consisted of conic sections, parabolas, hyperbolas and ellipses. Corresponding to conic sections are just two kinds of wavefronts, planar or spherical. The conic sections transform between different versions of either planar or spherical waves. For example, a parabola transforms an incident plane wave into a spherical wave centered on the focus. In radio astronomy the incident waves are always planar and the receiving feeds are pretty well matched to a spherical wave. So a single reflector antenna is always a parabola. Dual reflector antennas using conics consist of a parabola and either an ellipse or hyperbola which forms a new focus from the original focus of the parabola. In such dual reflector systems the intermediate focus between the two reflectors serves no direct purpose. This raises the very useful possibility of eliminating the intermediate focus to expand the range of possible reflectors. This memo will show two examples of shaped, dual offset reflector systems which demonstrates the much expanded range of possibilities available with this technique.

This memo is intended as a brief introduction to the range of solutions available from this particular shaping theory and software. The theory and mathematical techniques will be published in another set of memos. A very brief outline of the method can be found in [1]. There is a large body of literature devoted to dual shaping, references to this history will be given in other memos.

The two case studies both have a nominal aperture of 12 meters and each is approximately tailored to a particular feed. The first feed is German Cortes Quasi Self Complementary feed and the second feed is the log periodic ATA feed. In both cases, the geometry of the reflectors has been adjusted to accommodate the feed, minimize the cross polarization and keep the overall configuration mechanically sensible. The examples are both "Gregorian" type designs. The example feeds have wide opening angles (low gain) which make "Cassegrain" type designs impractical. These are introductory examples, not highly optimized designs. Achieving final optimized designs will require intensive interaction between ray optic designs, diffraction analysis with various feed data and a myriad of mechanical design considerations.

First example, the QSC feed
This first figure shows the ray trace of the reflector system in cross section. All of the reflector designs from the present software have a symmetry plane and this cross section is in that symmetry plane. The area of each reflector is shown in square meters. The entries under the square meter numbers are the reflector areas expressed as a fraction of the area of the 12 meter aperture. These two reflectors in combination have an area of about 1.6 times the aperture area.

The size of the subreflector is roughly estimated to be large enough to operate at 400 MHz. Diffraction analysis will be required to accurately define the required size of the subreflector. The right edge of the subreflector has a small clearance from the leftmost ray so the subreflector does not intercept much of the edge diffracted radiation from the primary.

One of the key elements of the design is the mapping of the rays from the focus to the aperture plane. In both of the example designs in this memo this mapping is chosen to be linear. The implication is whatever illumination that is provided by the feed is reproduced in the aperture to the limits of geometric optics. It must be emphasized that this mapping from the focus to the aperture is available as a design choice, many variations are possible. A mapping that takes a feed pattern with a deep edge taper and maps it to a more uniform aperture illumination with a quick rolloff near the aperture boundary is typical in many cases and is available as a choice here.
The QSC feed pattern requires a half angle at the subreflector edge of about 65 degrees, corresponding to an f/d of .39 and that is the half angle shown here. One of the challenges of this wide opening angle is avoiding reflected rays from the subreflector crossing close to the focus. Avoiding this requires pulling the subreflector down toward the left edge of the primary which indirectly pushes the right edge of the primary upward. This accomplishes the required clearance but does increase the primary surface area. This also slightly increases the value of the achievable minimum in cross polarization but not much (polarization discussed below). One advantage of this arrangement is the structure required to support the feed and subreflector is close to the back structure behind the primary.
The above cross section shows the rays in the symmetry plane cross section, the next figure shows where all the rays arrive in the aperture plane. The shaping method intrinsically maps circles of constant elevation angle at the focus to exact circles in the aperture plane. The "spokes" of constant azimuth at the focus can map to curved lines in the aperture except in the symmetry plane where they are always straight (symmetry plane vertical in the plot). The "spokes" in this aperture mapping are almost exactly straight everywhere because the geometric parameters have been adjusted, primarily the tilt angle of the boresight ray from the feed which is 41.5 degrees from vertical. Adjusting the geometric parameters to straighten the spokes also seems to minimize the cross polarization but that is empirical, there is no theoretical basis to precisely link the two. The linear aperture mapping is seen here as the constant spacing between radial points in each spoke.
The next figure shows an expanded view of the caustic region between the two reflectors in the symmetry plane. Spreading of the ray crossings is not too large in this case, the reflectors are not too different from best fitted conics which would have all the rays crossing at a point. Choosing a different aperture mapping will usually give a larger spreading of the ray crossing locus.
The next plot shows the two reflectors in 3D perspective. The cross section plot makes the secondary look relatively large. It looks a more sensible size in 3D, at least in comparison to the primary. The meshing shown on the two surfaces corresponds to the organization of the base data sets. The fundamental organization of the data is parametric in the spherical angles at the focus. For each equally spaced ray direction in the spherical angles there are x,y,z coordinates on the subreflector and primary corresponding to that ray. The number of points stored in the data sets can be adjusted up or down as needed, independent of the number of integration steps used in solving the partial differential equations.
The last plot shows the cross polarization in the aperture plane. The plot shows the y projection of a nominal unit x polarization. The vertical units in the plot are linear. The cross polarization is always zero in the symmetry plane which is front to back in the center of this plot. Whatever aperture taper is chosen for the design would multiply the values shown here, making the peak values near the rim smaller. The peak value in this plot is +.027 linearly or -31.3 db. The RMS of all the values is -43.8 db.

The linear cross polarization from this shaping method is always anti-symmetric in the aperture plane, regardless of the choice of geometric parameters. This means that the linear cross polarization always nulls to zero in the main beam direction. Choosing the geometric parameters to minimize the cross polarization across the aperture minimizes the cross polarization in all directions. A diffraction calculation is necessary to accurately predict the complete cross polar pattern.

Surface plot of the cross polarization
Second example, the ATA log-periodic feed

The text and figures in this second example will closely follow the sequence of the QSC example with appropriate modifications.

This first figure shows the ray trace of the reflector system in cross section. All of the reflector designs from the present software have a symmetry plane and this cross section is in that symmetry plane. The area of each reflector is shown in square meters. The entries under the square meter numbers are the reflector areas expressed as a fraction of the area of the 12 meter aperture. These two reflectors in combination have an area of about 1.34 times the aperture area, smaller than the QSC feed example. This demonstrates the possible effect of the feed opening angle on the overall reflector geometry.

The size of the subreflector is roughly estimated to be large enough to operate at 400 MHz. This subreflector is somewhat smaller than the QSC example based the fact it is less deeply curved. Diffraction analysis will be required to accurately define the required size of the subreflector. The right edge of the subreflector has a small clearance from the leftmost ray so the subreflector does not intercept much of the edge diffracted radiation from the primary.

One of the key elements of the design is the mapping of the rays from the focus to the aperture plane. In both of the example designs in this memo this mapping is chosen to be linear. The implication is whatever illumination that is provided by the feed is reproduced in the aperture to the limits of geometric optics. It must be emphasized that this mapping from the focus to the aperture is available as a design choice, many variations are possible. A mapping that takes a feed pattern with a deep edge taper and maps it to a more uniform aperture illumination with a quick rolloff near the aperture boundary is typical in many cases and is available as a choice here.

Reflector system cross section in the symmetry plane
The ATA feed pattern requires a half angle at the subreflector edge of about 42 degrees, corresponding to an f/d of .65 and that is the half angle shown here. The somewhat higher gain of the ATA feed allows more freedom in the location of the subreflector since self blockage of the reflected rays is less of a problem. The location shown here has the subreflector moved upward away from the left side of the primary which has the effect of moving the left edge upward and the right edge downward, thus reducing the primary reflector area. This location also slightly reduces the value of the minimum achievable cross polarization. The disadvantage of this is the larger distance from the feed/subreflector to the primary back up structure.

The above cross section shows the rays in the symmetry plane cross section, the next figure shows where all the rays arrive in the aperture plane. The shaping method intrinsically maps circles of constant elevation angle at the focus to exact circles in the aperture plane. The "spokes" of constant azimuth at the focus can map to curved lines in the aperture except in the symmetry plane where they are always straight (symmetry plane vertical in the plot). The "spokes" in this aperture mapping are almost exactly straight everywhere because the geometric parameters have been adjusted, primarily the tilt angle of the boresight ray from the feed which is 27.8 degrees from vertical. Adjusting the geometric parameters to straighten the spokes also seems to minimize the cross polarization but that is empirical, there is no theoretical basis to precisely link the two. The linear aperture mapping is seen here as the constant spacing between radial points in each spoke.
The next figure shows an expanded view of the caustic region between the two reflectors in the symmetry plane. Spreading of the ray crossings is even smaller than the QSC case, the reflectors are closer to being conics. Choosing a different aperture mapping will usually give a larger spreading of the ray crossing locus.
The next plot shows the two reflectors in 3D perspective. The cross section plot makes the secondary look relatively large. It looks a more sensible size in 3D, at least in comparison to the primary. The meshing shown on the two surfaces corresponds to the organization of the base data sets. The fundamental organization of the data is parametric in the spherical angles at the focus. For each equally spaced ray direction in the spherical angles there are x,y,z coordinates on the subreflector and primary corresponding to that ray. The number of points stored in the data sets can be adjusted up or down as needed, independent of the number of integration steps used in solving the partial differential equations.

Overall view of the reflectors
The last plot shows the cross polarization in the aperture plane. The plot shows the y projection of a nominal unit x polarization. The vertical units in the plot are linear. The cross polarization is always zero in the symmetry plane which is front to back in the center of this plot. Whatever aperture taper is chosen for the design would multiply the values shown here, making the peak values near the rim smaller. The peak value in this plot is +.01 linearly or -40.0 db. The RMS of all the values is -51.1 db.

The linear cross polarization from this shaping method is always anti-symmetric in the aperture plane, regardless of the choice of geometric parameters. This means that the linear cross polarization always nulls to zero in the main beam direction. Choosing the geometric parameters to minimize the cross polarization across the aperture minimizes the cross polarization in all directions. A diffraction calculation is necessary to accurately predict the complete cross polar pattern.
Conclusion

The two case studies presented give an introduction to this dual reflector shaping method. There are several key features. The mapping from the focus to the aperture plane can be selected to optimize the design for the specified task. The linear mapping in the examples is just the simplest of many possible choices. The aperture always consists of concentric exact circles spaced according to the chosen mapping. The method can accommodate a wide range of feed opening angles (f/d) and still minimize the crosspolar distortion in the aperture. These features give a much wider range of possible reflector designs than conic sections.

This shaping software has been self tested in its present form and past versions have produced reliable results. However, the next step should be to verify the reliability of the software by analyzing several examples with physical optics. These would not have to be final designs, just examples of potential geometries. The initial analysis should use a perfect, theoretical feed to isolate just the behavior of the reflector designs. Particularly, the predicted cross polarization behavior should be verified. It would be interesting to analyze one or more examples with poor cross polarization to see the behavior over a range of values. One other result of initial analysis would be better estimates of the subreflector size required for good low frequency performance.

Once confidence in the method has been established, this technique can be the source of multiple designs, each adapted a particular candidate feed and diffraction analyzed with measured feed data. Each of these designs would also be studied for cost and mechanical feasibility. This would give a set of choices moving forward to selection of a prototype antenna to be constructed.