Memo 2

US SKA Technology Development Project
Memo Series

Survey Metrics

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v1.4, January 2009

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October 17,2007, Rev December 4, 2007, Rev January 9, 2009
Version 1.4

Introduction

Much of the SKA's science case involves large-scale surveys that pose particular and extreme demands on the system design. However, the different kinds of surveys (blind, multiple targets, continuum, spectral line, time domain) and the recognized need for deep integrations on small numbers of targets have conflicting requirements. This document presents a variety of metrics that can be used for optimizing peformance against design. Some of these were considered in SKA Memos 66 and 85[1]. *Etendue*, the product $A\Omega$ is often used as a figure of merit for telescopes using the same electromagnetic band (especially optical and IR) but is useless for comparing the survey throughputs of different kinds of telescopes (e.g. radio and optical) observing the same target sources (e.g. galaxies). A common metric — survey speed — is useful for considering survey throughput, but it is typically considered for steady, continuum sources and without consideration of statistical significance in the detection scheme, which must consider the number of trials made in a survey and which differs for continuum and spectral line surveys. Survey speed as usually defined does account for source variability, which can be extreme for transient sources, of great current interest for the SKA and its precursors. These additional issues are considered here. Detailed derivations are given in a separate document.[2] It should be emphasized that even the more general metrics presented here are applicable only to source populations distributed homogenously in Euclidean space. Specific surveys should take into account whether Galactic or cosmological lines of sight are being considered.

Survey Metrics

In the following we assume that a survey is conducted by sampling in a time T_s a total solid angle Ω_s that is much larger than the instantaneous solid angle sampled by the telescope, Ω_i. Sampling may be via an explicit raster scan or by point-and-dwell observations on a grid of sky positions. The integration time per sky position τ is roughly $\tau = T_s(\Omega_i/\Omega_s)$. For simplicity we assume that the gain is uniform across the sampled field of view. A more precise treatment of a particular survey would need to include non-uniform gain. The survey metrics presented here are blunt instruments, useful for comparing telescopes at a level that ignores how surveys are conducted, but for designing surveys themselves.

I. Steady Sources

Survey Speed: The survey speed is

$$\mathrm{SS} = \frac{\Omega_i}{\tau} = \frac{B S_{\min}^2}{2k^2}\left(\frac{N_{\rm FoV}\Omega_{\rm FoV}}{N_{\rm sa}}\right)\left(\frac{f_c A_e}{m T_{\rm sys}}\right)^2 \quad (1)$$

where

$\Omega_i \equiv N_{\rm sa} N_{\rm FoV}\Omega_{\rm FoV}$

$\Omega_{\rm FoV}$ = field of view, the size of one single-dish pixel

$N_{\rm FoV}$ = number of fields of view (i.e. the number of pixels in a multiple-feed cluster or phased-array feed)

$N_{\rm sa}$ = number of subarrays into which the collecting area is divided, assumed to be equal sized, with consequent reduction in gain and increase in instantaneous field of view

B = bandwidth

$S_{\min}$ = minimum detectable flux density given by (assuming two polarization channels are summed)

$$S_{\min} = \frac{m \times \mathrm{SEFD}}{\sqrt{2B\tau}} = \frac{2m\, k T_{\rm sys} N_{\rm sa}}{A_e\sqrt{2B\tau}} \quad (2)$$

SEFD = system equivalent flux density of a subarray

m = threshold signal-to-noise ratio

A_e = effective area of the *entire* array

f_c = fraction of A_e that is usable in the survey, e.g. antennas in the core

$T_{\rm sys}$ = system temperature

[1] SKA Memo 66 *Relative Sensitivities of the SKA and Other Current or Near-Future Instruments*, 2005, D. Jones; SKA Memo 85, *Discovery and Understanding with the SKA* (Appendix A), 2006, J. Cordes et al. www.skatelescope.org/pages/page_astronom.htm

[2] SKA Memo 97, *The SKA as a Synoptic Survey Telescope: Widefield Surveys for Transients, Pulsars and ETI*

k = Boltzmann's constant.

As defined, SS has proper physical units (solid angle per unit time). However, a more useful form dispenses with some of the constants, leading to a figure of merit based on survey speed:

$$\mathrm{FoMSS} = B\left(\frac{N_{\mathrm{FoV}}\Omega_{\mathrm{FoV}}}{N_{\mathrm{sa}}}\right)\left(\frac{f_c A_e}{m T_{\mathrm{sys}}}\right)^2. \tag{3}$$

This expression is consistent with the common form often used, FoV $(A_e/T_{\mathrm{sys}})^2 B$, (sometimes without the bandwidth factor) but makes explicit for SKA applications that only the core array is usable for some surveys and that the survey speed depends on the significance level, m. It also shows the explicit dependence on number of fields of view and subarrays. It is clear that $N_{\mathrm{sa}} = 1$ maximizes survey speed. However, there are cases where instantaneous sky coverage may override sheer survey speed, such as the case where very rare, fast transients are sought (see below).

Optimizations that fix the survey speed FoMSS must consider tradeoffs between field of view, $(A_e/T_{\mathrm{sys}})^2$ and processed bandwidth.

Volume Surveyed: The number of sources detected is proportional to the volume surveyed (V_s) for a population of sources distributed homogeneously.. Quantifying V_s leads to a survey metric that is related to FoMSS. Our analysis applies to Euclidean space. Consider a raster-scan type survey, as before, where the total solid angle Ω_s and total time T_s to scan it are held fixed. The volume surveyed is $V_s = \frac{1}{3}\Omega_s D^3_{\mathrm{max}}$, where $\Omega_s = 4\pi f_{\mathrm{sky}}$ is the total solid angle surveyed, taken to be a fraction f_{sky} of the entire sky, and $D_{\mathrm{max}} = (L_p/S_{\mathrm{min}})^{1/2}$ is the maximum distance reached for a minimum detectable flux density, S_{min}, as before, and for a pseudo-luminosity $L_p = SD^2$ for a source of flux density at some fiducial distance D. Formally, we have

$$V_s = \frac{1}{3}\left(\frac{L_p}{2k}\right)^{3/2}\Omega_s^{1/4}\left[T_s \times \mathrm{FoMSS}\right]^{3/4}, \tag{4}$$

with FoMSS defined as above. Alternatively, if the survey depth D_{max} is held fixed along with the total survey time, the volume surveyed is linear in FoMSS, $V_s \propto$ FoMSS (as in Memo 85).

Discussion: We conclude that FoMSS is an adequate survey metric that follows from consideration of either the rate at which solid angle is covered or volume is sampled. As already stated, it is assumed that sources are uniformly distributed in Euclidean space; in addition the simple form of the survey metric assumes that they are steady. For populations that are spatially confined, such as pulsars in the Galaxy or galaxies in the Virgo cluster, the rate of source detection depends differently on array parameters.

Detection rate: The number of source detections is related to the product FoMSS $\times\, T_s$, where T_s is the total survey time; covering more sky through an increase in T_s increases the yield. However, at some point, the numbers will saturate because we run out of sky. For example, we may survey just the Galactic plane or we may wish to survey the accessible portion of the entire sky. When saturated, the only recourse for increasing the number of detections is to integrate longer per sky position or increase the bandwidth B for continuum sources. The survey yield then scales more slowly with T_s. For steady sources, the tradeoff can be made between field of view and sensitivity, but is straight forward only in the linear regime before solid-angle saturation.

Number of trials: Different kinds of surveys imply different significance levels m that should be taken into account when using the survey metric. Here we consider example surveys that cover the entire bandwidth B, including continuum and spectral line surveys with varying spectral resolution.

1. *Continuum Sources:* For a continuum survey with processed bandwidth B, source detection involves one statistical trial per pixel if no channelization is used in the detection analysis. The significance level m is then determined by some appropriate false alarm rate that takes into account noise and confusion.

2. *Doppler Broadened line Sources:* For a Doppler velocity width ΔV, the number of channels and thus the approximate number of trials per pixel is $N_{\mathrm{trials}} \sim (c/\Delta V)\ln(1 + B/\nu) \approx (cB/\nu\Delta V)$, where ν is the lowest frequency in the band and the approximate equality holds for $B \ll \nu$. For an HI survey with $\Delta V = 100$ km s^{-1}, $\nu = 0.5$ GHz and $B = 0.5$ GHz, we have $N_{\mathrm{trials}} \approx 2100$.

3. *SETI with constant channel width:* SETI searches with channel bandwidths $\Delta\nu_{\mathrm{ch}} \lesssim 1$ Hz yield per pixel $N_{\mathrm{trials}} = 10^9(B/1\ \mathrm{GHz})(1\ \mathrm{Hz}/\Delta\nu_{\mathrm{ch}})$.

4. *Pulsar survey:* Pulsars with approximately steady pulse amplitudes may be considered here. Blind surveys require dedispersion with a set of trial dispersion measures (DMs), comparable to the number of channels across the band. The channel width is related to the target time resolution in the survey. Statistical tests are done on the power spectrum of the time series for each trial DM. The number of trials is the product of the number of trial DMs (typically thousands) and (half) the FFT length (millions), multiplied by the number of harmonic sums investigated

per FFT bin (tens) and, if applicable, the number of acceleration values (hundreds to thousands). Without an acceleration search, the number of trials per pixel requires a large S/N threshold, e.g. $m = 8$. Acceleration searches require thresholds larger than those of SETI searches. See below for a figure of merit that applies to surveys of steady pulsars.

The number of trials per pixel varies greatly between these types of surveys, requiring different detection thresholds for fixed false-alarm probability.

II. Intermittent Sources

Surveys of time-variable sources are challenging because there is great diversity in event rates, durations and source density. One extreme is the canonical pulsar population, which is relatively dense in the Galactic disk with large event rates. Another comprises (potential) prompt radio bursts from gamma-ray burst sources, which occur in gamma-rays at a rate ~ 1 day^{-1} hemisphere^{-1}. These cases require radically different specifications in order to provide adequate sampling.

A metric that includes source intermittency can be built around the probability that an emission event occurs when a source is pointed at. We assume that the survey is conducted in raster-scan mode. Assuming Poisson statistics, the effective volume that is sampled is

$$V_{\rm s,variable} \propto V_{\rm s,steady} \left(\frac{\tau_{\rm eff}}{\tau}\right)^{3/4} P_t(\eta, W, \tau), \tag{5}$$

where $\tau_{\rm eff} = \tau$ for steady sources or slow transients with $W \gg \tau$, while $\tau_{\rm eff} = W$ for fast transients. As an approximation we write $\tau_{\rm eff} = (W^2 + \tau^2)^{1/2}$. The temporal probability factor is

$$P_t(\eta, W, \tau) = P_{\geq 1} = 1 - e^{-\eta (W^2+\tau^2)^{1/2}}, \tag{6}$$

the probability of having one or more emission events occur from a given source in the time τ. The term in the exponential accounts for an event partially overlapping with the time interval τ in which the source is in the overall field of view. Note that this reckoning counts *sources* not events. In Memo 97 a related expression for event counts is given.

In a raster scan of total solid Ω_s made with an instantaneous solid angle Ω_i over a total time T_s, the time spent per sky position is $\tau = T_s \Omega_i / \Omega_s$, as before. The number of *detectable* sources is given by the number of sources in the volume that show events at any time during the scan time T_s. The number of actual detections depends on if a given source is pointed at and whether transient events are bright enough. P_t takes into account the pointing aspect while the volume searched takes into account the latter.

We now define two metrics for transient sources.

Survey Metric for Transients: First, we extend FoMSS by noting that FoMSS $\propto V_s^{4/3}$ so the effective volume of Eq. 5 suggests a FoM for transient sources

$$\text{FoMTS} \propto N_{\rm det}^{4/3}(TS) \propto N_{\rm det}^{4/3}(SS) \left(\frac{\tau_{\rm eff}}{\tau}\right) P_t^{4/3}(\eta, W, \tau), \tag{7}$$

or

$$\text{FoMTS} = \text{FoMSS} \times K(\eta W, \tau/W) \tag{8}$$

$$K(a, x) = \left(1 + x^2\right)^{-1/2} \left[1 - e^{-a(1+x^2)^{1/2}}\right]^{4/3} \tag{9}$$

where $a \equiv \eta W$ is the product of event rate per source η and event duration W, which is a measure of the overlap of events in time, and $x \equiv \tau/W = \Omega_i/\dot{\Omega} W$. The quantity $\dot{\Omega} = \Omega_s/T_s$ is the mean rate at which solid angle is surveyed. The K function is plotted in Figure 1 and is discussed further in Memo 97, Appendix A.

Completeness Coefficient for Source Detection: We define a *completeness coefficient* as the ratio of the expected number of sources that *are* detected to the number of sources in the population that display one or more temporal events (if variable) during a survey scan:

$$C_s = \frac{N_{\rm det}(\tau)}{N_{\rm obj}(T_s)}. \tag{10}$$

where $N_{\rm det}$ is the number of sources *actually detected* in a survey while $N_{\rm obj}$ is the number of sources that are *detectable*, i.e. that emit a burst during a survey, whether or not they are pointed at during the event. C_s measures not just the volume that a survey samples, but also the likelihood that a source is "on" during the dwell time that it is pointed at. For a homogeneous population of sources, the completeness coefficient becomes

$$C_s = \frac{\Omega_s}{\Omega_{\rm pop}} \left(\frac{D_{\rm max}}{D_{\rm pop}}\right)^3 \frac{P_t(\eta, W, \tau)}{P_t(\eta, W, T_s)}. \tag{11}$$

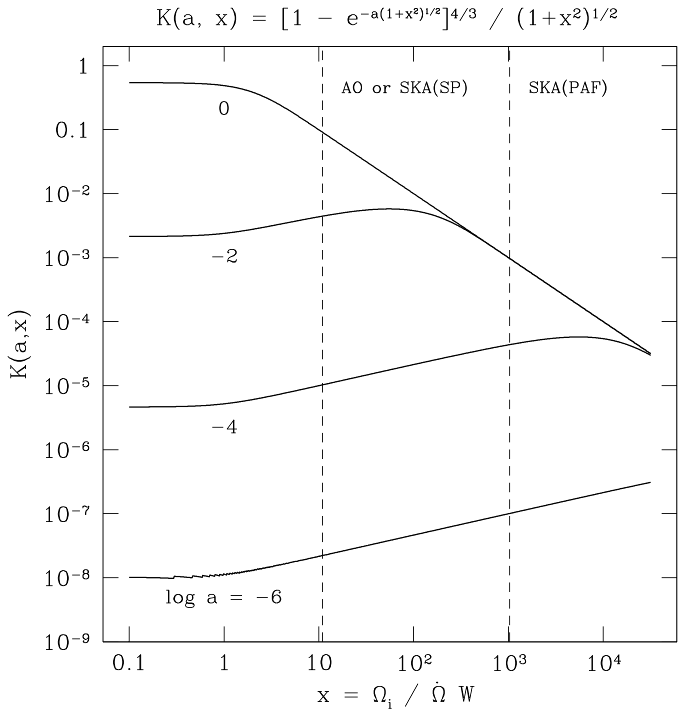

FIG. 1.— Plot of the temporal factor $K(a,x)$ defined in Eq. 9 for several values of a. The vertical lines denote values for $x = \Omega_i T_s/(\Omega_s W)$, where Ω_i is the instantaneously sampled solid angle, Ω_s is the total solid angle surveyed in time T_s, and W is the transient duration. We assume $W = 1$ s in both cases. The leftward line applies to an extragalactic survey using the 7-beam ALFA system with 3.5 arcmin beam widths at Arecibo that surveys 30% of the sky in 2000 hr. The same x is achieved for an SKA system with single-pixel feeds having 1 deg beams that survey 80% of the sky in 5 days. The line labelled "SKA(PAF)" is for the same SKA survey but using a phased-array feed with 100 beams.

$D_{\rm max}$ here is calculated using the correct effective integration time, $\tau_{\rm eff}$. For steady sources or variable sources with $\eta(W^2+\tau^2)^{1/2} \gg 1$ the temporal probability $P_t \to 1$ and C_s reduces to a simple volume ratio. For variable sources that have "low" rates such that $\eta(W^2+\tau^2)^{1/2} \ll 1$, we have $P_t \approx \eta(W^2+\tau^2)^{1/2}$ and there are two regimes $W \ll \tau$ and $W \gg \tau$. These are considered elsewhere for specific source classes.

The completeness coefficient defined here counts *sources*. Another can be defined that counts *events*, yielding a different form. Details can be found in Memo 97 cited on the first page.

Discussion: Both the FoM for transient sources and C_s put a premium on instantaneous solid angle, especially for fast, low rate transients i.e. those with short durations W and small values of η.

Both FoMTS and C_s have nontrivial scalings with W and η owing to there being different regimes in the combination of η, W and τ.

III. Pulsars ("Steady Transients")

Many pulsars, though time dependent, have relatively steady pulse amplitudes. A survey metric for pulsars therefore bears some resemblance to that for steady sources, but the detectability is a function of period P and dispersion measure DM.

The survey volume is $V_s = \frac{1}{3}\Omega_s D_{\rm max}^3$ with $D_{\rm max} = (L_p/S_{\rm min})^{1/2}$ as before, but now the minimum detectable flux density is written as

$$S_{\rm min}(P,{\rm DM}) = \frac{S_{\rm min_1}}{h_\Sigma(P,DM)}, \tag{12}$$

$$S_{\rm min_1} = \frac{2m\,kT_{\rm sys}N_{\rm sa}}{A_e\sqrt{2B\tau}}, \tag{13}$$

where the subscript "1" on $S_{\rm min_1}$ denotes the threshold for a single harmonic. The harmonic sum h_Σ is given by

$$h_{\sum}(P,{\rm DM}) = {\rm N_h^{-1/2}} \sum_{\rm j=1}^{\rm N_h} \left|\tilde{\rm f}_{\rm j}\right|, \tag{14}$$

where f_j is the Fourier amplitude for a time series dedispersed using dispersion measure DM; the harmonic number j corresponds to harmonics j/P of the period P; f_j is normalized to the zero-frequency ($j=0$) value and N_h is the number of harmonics that maximizes the sum; h_Σ is equal to unity if the signal is an undistorted sinusoid but can be much larger than unity when the sum is optimized for $N_h \gg 1$. For a gaussian-shaped pulse, this number is $N_h \approx P/2W$ for a period P and pulse width (FWHM) W. If the pulse is heavily broadened by instrumental effects, scattering or orbital motion, $h_\Sigma \ll 1$ and the survey becomes insensitive.

Using these definitions, the figure of merit for a pulsar survey is

$$\begin{aligned} \mathrm{FoMPSR} &= \mathrm{FoMSS} \times h_\Sigma^2(P, DM) \\ &= B \left(\frac{N_{\mathrm{FoV}} \Omega_{\mathrm{FoV}}}{N_{\mathrm{sa}}}\right) \left(\frac{f_c A_e}{m T_{\mathrm{sys}}}\right)^2 h_\Sigma^2(P, DM). \end{aligned} \quad (15)$$

Comments:

1. $h_\Sigma(P, \mathrm{DM})$ is dimensionless and is simply a multiplier of the steady-source FoM.
2. Propagation effects (dispersion and scattering) reduce h_Σ for high-DM objects because pulses are smeared out, particularly at low frequencies; $h_\Sigma(P, \mathrm{DM})$ is therefore strongly frequency dependent. Coherent dedispersion can remove all dispersion smearing and restore pulses or bursts to their pre-propagation form. Scattering smearing cannot be removed but it can be addressed in matched-filter template banks designed to optimize signal-to-noise ratios in searches.
3. Application of Eq. 15 is best done by averaging over direction, e.g. Galactic coordinates ℓ, b, for a particular survey.
4. As presented, h_Σ and FoMPSR are explicit functions of P and DM. One could alternatively define the independent variables as being P and distance, D. The corresponding DM will vary greatly for different directions, in this case. In the end it probably makes little difference as to whether DM or D is used once an average over direction is done.
5. The pulsar FoM, like the other FoMs, implicitly is based on the assumption that the population of sources is homogeneously distributed. Careful application of FoMPSR must recognize the spatial distribution of pulsars in the Galaxy and that highly sensitive surveys will reach the boundaries of the Galactic population. However, they may also reach extragalactic populations.