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Diffraction Analysis of
a Preliminary Dual Shaped Reflector Design
for the SKA/TDP Prototype Antenna

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Introduction

This memo presents the results from diffraction analysis of a dual reflector design intended as an early concept for the SKA/TDP prototype antenna. The analysis was performed by Christian Holler using GRASP9, a commercial antenna analysis software package. The antenna geometry is a dual shaped design from custom software developed by Lynn Baker. This first analysis is meant to verify the shaping software and provide a first look at the performance of antennas of the general size and geometry anticipated for the SKA/TDP prototype. The analysis does not use feed data from any of the broad band feeds presently under development. An idealized feed defined inside GRASP9 which has a gaussian pattern is used to illuminate the reflectors. Using this idealized feed at this early stage isolates the performance of the reflectors from any real feed behavior, providing insight into how the reflectors perform by themselves. This is actually a good strategy overall, use both an ideal feed and then a real feed to separate the effects of the reflectors from the feed behavior.

The reflector geometry is shown in cross section in Figure 1. The plane of the figure is the symmetry plane of the reflectors. The basic configuration is very similar to a design presented in a previous memo [1]. The reflectors are 3.0 m. and 14.5 m. rim to rim in this cross section. The feed opening half angle is 42° and the feed boresight is tilted -28.2° from the z axis. The feed opening angle is the same as used for the ATA log periodic feed.

The reflectors are designed using geometric optics to map the bundle of rays from the feed to the aperture in a prescribed fashion. In this design the chosen form of the mapping is linear from elevation angle at the feed to radial distance in the aperture. Dual reflector shaping is typically associated with a mapping which maximizes the signal to noise ratio (G/T) of the system. Such a mapping is quite non-linear in contrast to this design. It must be emphasized that there are many possible choices for the mapping from the feed to the aperture, each with its own set of advantages and drawbacks.

The aperture mapping is shown in Figure 2, the circles are exactly concentric which is intrinsic to the shaping theory and are equispaced which results from the chosen mapping. The “spokes” which correspond to equal azimuth values at the feed are almost exactly straight. This is achieved by adjusting the feed tilt angle (boresight) to an optimum angle which straightens the spokes and also minimizes the geometric cross polarization. This tilt is highly analogous to the Mizugutch condition which minimizes the cross polarization in conic section designs. The effect of choosing this linear mapping is the feed pattern illuminating the secondary reflector is reproduced in the aperture within the limits of geometric optics. In particular, the edge taper on the secondary is approximately reproduced in the aperture.

Parameters of the Diffraction Analysis

The idealized feed used in the analysis has a gaussian beam shape and the chosen edge taper on the secondary reflector is -10 db. The choice of -10 db. is a “typical” number for prime focus antennas with moderate noise receivers and this choice will be reconsidered in future analysis. The feed is linearly polarized with the electric field in the x direction on boresight, leading to a nominal x oriented polarization across the aperture. This polarization corresponds to Ludwig’s third definition. The feed representation accounts for near field behavior where the reflectors are not in the far field of the feed. Appendix 1 has some material adapted from the GRASP9 manual describing this ideal feed.

The analysis inside GRASP9 includes the usual physical optics calculation augmented by physical theory of diffraction to calculate edge effects. The far field results from the analysis are presented in absolute gain relative to isotropic (dbi).
The mapping from the focal point to the aperture is linear, the radius in the aperture is linear in elevation angle at the focus. The area of each reflector is shown in square meters, the numbers beneath are the reflector areas as a fraction of the aperture area. The plane of the drawing is x-z, z is upward, x is to the right and y is RH, into the page. The caustic between the reflectors is not a point, see [1] for a close up graphic.
The linear mapping from the focus to the aperture is obvious in this plot. The tilt angle of the feed has been selected to keep the “spokes” almost perfectly straight. This also minimizes the geometric cross-polarization. The linear mapping implies that the feed pattern (in particular, the edge taper) at the focus is approximately reproduced in the aperture.
Far Field Plots, Wide Angle Behavior

Figures 3, 4, 5 show the far fields for three azimuth cuts, all elevation angles and three frequencies. The spherical coordinates are the standard angles attached to the coordinate system shown in Figure 1. The plane of the figure is the $\phi=0^\circ$ cut. These plots are intended to display the wide angle behavior of the antenna system, the critical central area will be shown in closer detail in other plots. The choice of frequencies, 1 GHz., 500 MHz. and 250 MHz., is intended to investigate the performance at long wavelengths. Both the copolar and crosspolar fields are shown. The various plots are color coded to the particular $\phi$ cut and the polarization. The cross polarization is zero in the $\phi=0$ cut (symmetry plane) and does not appear. The vertical scale is absolute gain in db. relative to isotropic.

The general features at wide elevation angles are quite consistent at the three different frequencies with a general trend of getting larger as the frequency decreases. With a few exceptions, everything is below isotropic and the exceptions are mostly in the $\phi=0$ cut. Figure 6 shows the $\phi=0$ cut at all three frequencies. The consistency of the pattern behavior is quite apparent.

There is an interesting feature in the range of roughly $\theta=+100^\circ$ to $+180^\circ$ in the $\phi=0$ cuts, there is an interference pattern with its largest peak around $110^\circ$. From this view angle only the back of the primary reflector is visible so the source of this feature must be scattering from opposite edges of the rim of the primary going in and out of phase as the view angle changes. This especially apparent in Figure 6 where the ripple spacing halves for each frequency doubling.

There is also a consistent feature around $\theta=+75^\circ$ which rises above isotropic. This $\theta$ value corresponds very closely with the direction of the ray from the left rim of the secondary to the right rim of the primary so this feature is likely due to secondary to primary spillover.

There is another feature above isotropic around $\theta=-25^\circ$ which is directly behind the secondary. This may well due to edge diffraction from the secondary rim, similar to the primary.

One way to better understand the origin of various wide angle behavior would be to produce these results with and without the edge diffraction calculation included.
The far fields at 1 GHz in three azimuth cuts and all elevation angles. The red, green, blue traces are the copolar fields and the purple, brown traces are the cross polar fields. Except for two small spots in the red trace, everything outside of the vicinity of the main beam area is below isotropic.
The far fields at 500 MHz in three azimuth cuts and all elevation angles. The red, green, blue traces are the copolar fields and the purple, brown traces are the cross polar fields. Except for two small spots in the red trace, everything outside of the vicinity of the main beam area is below isotropic.
Figure 5

250 MHz.

The far fields at 250 MHz in three azimuth cuts and all elevation angles. The red, green, blue traces are the copolar fields and the purple, brown traces are the cross polar fields. At this low frequency there is more of the red trace above isotropic and a small area of the cross polar field is above isotropic. There is some distortion of the main beam in the azimuth zero cut.
The far fields at all three frequencies in the azimuth zero cut and all elevation angles. The wide angle fields are mostly below isotropic in this cut with a few exceptions. The general trend is for the wide angle fields to get larger as the frequency decreases. The overall location and type of features is fairly consistent with frequency.
Far Field Plots, Main Beam, Near Sidelobes, Cross Polarization

Figures 7, 8, 9 show the main beam, the near sidelobes and the cross polar lobes at the three frequencies. Note that the horizontal scale in each plot doubles in 0 span, keeping the apparent width of the main beam roughly the same. The vertical scale is adjusted to the peak of the main beam. The coloring coding of the cuts is the same as before. The peak level of the first sidelobe and and cross polar lobe are labeled with absolute gain in dbi. and gain relative to the peak of the main beam in db.

The gain varies approximately as the expected -6db. for halving the frequency. There is also some main beam offset and distortion as the frequency drops. More detail of this is shown in the next section. The first sidelobe is consistently about -23 db. below the main beam which fits with the chosen -10 db. edge taper from the feed and in the aperture plane. Note that this means, as the frequency gets smaller, the sidelobes get smaller in absolute gain and closer to isotropic where the wide angle clutter starts. At 1 GHz, the first sidelobes are fairly symmetric in all three cuts and fairly distinct second sidelobes are present. As the frequency drops lower, the first sidelobes become less symmetric. At 250 MHz, the first null on the positive 0 side has filled in and the first sidelobe has become part of the main beam. Figure 10 shows all three frequencies in the $\phi=0$ cut. The roughly -6db. steps in gain and the doubling of the beamwidth is clear. The distortion of the main beam with decreasing frequency is evident.

The cross polarization is zero in the symmetry plane and peaks in the $\phi=90^\circ$ plane. The cross polarization is about constant for all three frequencies at roughly +8 dbi. This means that the size of the cross polarization relative to the main beam gets bigger as the frequency decreases and the gain of the main beam decreases. The relative size of the cross polarization is approximately -32 db. at 1 GHz. but decreases to -26 db. and then -20 db. as the frequency goes down.
Figure 7

A closer view of the main beam, near in sidelobes and crosspolar lobes at 1 GHz. The cross polarization is zero in the azimuth zero cut and peaks in the cut perpendicular to azimuth zero. The sidelobe and cross polar lobes are labeled with dbi. and db. relative to the peak of the main beam. There is a tiny amount of main beam distortion.
A closer view of the main beam, near in sidelobes and crosspolar lobes at 500 MHz. The cross polarization is zero in the azimuth zero cut and peaks in the cut perpendicular to azimuth zero. The sidelobe and cross polar lobes are labeled with dbi. and db. relativie to the peak of the main beam. The distortion in the main beam has grown slightly as the frequency decreases.
A closer view of the main beam, near in sidelobes and crosspolar lobes at 250 MHz. The cross polarization is zero in the azimuth zero cut and peaks in the cut perpendicular to azimuth zero. The sidelobe and cross polar lobes are labeled with dbi. and db. relative to the peak of the main beam. The distortion in the main beam is now more pronounced with a significant shift in the peak direction.
Figure 10

A closer view of the vicinity of the main beam at all three frequencies in the azimuth zero cut. The expected decrease in gain and increase in beamwidth is obvious. There is some distortion in the 250 MHz beam on the right side.
Far Field Plots, Central Portion Of the Main Beam

Figures 11, 12, 13 show the center portion of the main beam at the three frequencies and three azimuth cuts. The horizontal axes are doubled in span as the frequency is halved, keeping the apparent width about the same. The same color coding applies to the traces and the half beamwidths shown at the -3 db. level.

The peak gain and corresponding aperture efficiency are shown on each plot. The gain and efficiency numbers include feed spillover, spillover between the reflectors and all amplitude and phase effects in the aperture. The efficiency at 1 GHz. is close to the value approximated from geometric optics. The feed taper of -10 db. corresponds to roughly 15% spillover, an efficiency factor of .85. Assuming the same edge taper to apply across the aperture gives a taper efficiency of about .85 as well. Combining those terms predicts .72 which is very close to the value from GRASP9. At the lower frequencies, the efficiency falls off, dropping to .66 at 500 MHz. and down to .56 at 250 MHz. Given the small size of the secondary reflector at 250 MHz. (~2.5 wavelengths), .56 efficiency is actually quite good.

The 3 db. half beamwidths show a pointing offset in the peak of the main beam. This offset is in the symmetry plane and gets larger as the frequency decreases, starting at .05° at 1 GHz. and increasing to .16° at 500 MHz. and to .54° at 250 MHz. Note that this increase is larger than the expected doubling in beamwidth as the frequency halves. There is also some distortion of the beamshape. Part of this is due to the ϕ=45°, 90° cuts being attached to the z axis and not the direction of the main beam peak. If a new z axis was defined in the peak direction and the cuts recalculated, some, but not all of this distortion would disappear.

The pointing offset is due to a linear phase tilt across the aperture. The most likely source of this phase tilt is near field effects between the feed and the secondary. At 250 MHz. there is only ~1.4 wavelengths between the focus and the left rim of the secondary and ~2.3 wavelengths to the right rim. This places the secondary well into the near field of the feed where the phase fronts from the feed are not spherical. (See appendix I.) The variability of the illumination across the secondary from near field effects could also explain some of the other features in the far field patterns. Despite the distortions of the main beam at 250 MHz., the performance is not that bad once the pointing offset is removed.
A very close view of the top of the main beam at 1 GHz. The -3 dB beamwidths are color coded to the traces for each azimuth cut. There is a very small shift in the main beam peak direction. The beamwidths are almost identical in all three cuts.
A very close view of the top of the main beam at 500 MHz. The -3 db. beamwidths are color coded to the traces for each azimuth cut. There is a modest shift in the main beam peak direction. The beamwidths are very similar in all three cuts.
A very close view of the top of the main beam at 250 MHz. The -3 db. beamwidths are color coded to the traces for each azimuth cut. The shift in the main beam peak direction is more pronounced at this frequency. The beamwidth variation is more pronounced but this is partly due to the azimuth cuts not being centered on the main beam peak direction.
Conclusions

The preliminary design and analysis presented here verifies the performance of the shaping software and its interface to GRASP9. The design itself performs mostly as expected for the parameters chosen. The one interesting result is the good performance at 250 MHz, where the subreflector is electrically quite small and in the near field of the idealized feed. This is probably due in part to the chosen linear mapping which keeps the reflectors smooth. In particular, the radius of curvature of the secondary does not vary too rapidly.

The wide angle behavior is consistent with the qualitative view of spillover and edge diffraction being proportional mostly to edge illumination intensity and slowly getting larger as the frequency is reduced. Since wide angle response increases slowly with frequency and the main beam gain decreases 6 db. per octave of frequency change, the wide angle response relative to the main beam decreases a little more than 6 db. per octave. The ratio of wide angle response to main beam gain is essentially the rejection ratio of wide angle sources or interference. This important performance parameter will vary with frequency, and get steadily worse at low frequencies.

There is a caveat about the details of the wide angle response. The exact response depends on the mechanical details at the rims of the reflectors. The surfaces defined in GRASP9 are ideal mathematical surfaces and do not have depth or structure as presently defined.

Reducing the wide angle response will require reducing the edge illumination. If large reductions in wide angle response are required then very deep edge tapers will be required. In the context of a linear mapping as shown here, this will lead to very low aperture efficiencies. It is possible to choose a mapping which restores the aperture efficiency while keeping the edge tapers very low but this requires a highly non-linear mapping which results in the secondary having a widely variable radius of curvature. This will sharply degrade the low frequency performance seen in the design analyzed here.

The shaping method produces an anti-symmetric cross polarization in the aperture which is zero along the symmetry centerline of the aperture. This produces a null in the far field cross polarization along the \( \phi=0^\circ \) azimuth cut regardless of the geometric parameters. Optimizing the feed tilt angle minimizes the cross polarization across the whole aperture. The far field cross polarization is maximum in the \( \phi=90^\circ \) plane perpendicular to the symmetry plane. In this design the cross polarization peaks at about +8 dbi. for all three frequencies. The peak occurs inside the main beam at very roughly the -6db. level. Since the cross polarization is roughly constant with frequency its value relative to the main beam increases as the frequency goes down and the gain is reduced.

Appendix 1

From the GRASP9 Reference Manual:

The *Gaussian Beam, Pattern Def.* provides a beam with a Gaussian taper. The beam is radiating along 0=0° in the feed coordinate system specified by *coor_sys* (the x, y, z -coordinate system).

The beam is broader in the near field than in the far field. It is a closed form model satisfying Maxwell’s equations in both near and far field. Because of the taper, the behaviour of the near field is more complicated than that of a point source, so that the phase fronts are not simply spherical, but rather ellipsoidal. The phase centre for the far field is at the waist of the Gaussian beam which is at the origin of the feed coordinate system.

The feed pattern is normalised to dBi, i.e. to radiate 4 π watts. The total radiated power will be 4 π times the value of *factor* (converted from dB to power).

The pattern is not expanded into spherical waves as the model includes a precise near field description.